

# Welcome to Algebraic Reasoning Content Academy

- A number of different graphs are posted around the room. Using a colored dot, put “yourself” on each graph.
- Introduce yourself to the people at your table.



# Algebraic Reasoning Content Academy—Grade 5 Round Rock ISD

Learning by Doing®

## Sami Briceño

# Content Academy Goals

Teachers will have the opportunity to:

- Increase their content knowledge of algebra and algebraic reasoning
- Connect upper elementary patterns and algebraic reasoning in vertical alignment with middle and high school algebra
- Practice assessing the cognitive level and rigor of algebra/pattern tasks and consider strategies to increase and maintain high cognitive level
- Receive an intro to the new Algebra Kits and connect kit lessons to topics in training



# Learning Outcomes—Day 1

Aligning teaching to the way students learn

- Solve, analyze, and discuss mathematical tasks
- Consider the effects of different levels of mathematical tasks on students' achievement
- Investigate patterns of numbers in hundred, fifty and eighty board
- Explore Hundred Board activities across grade levels 3, 4, and 5
- Extend hundred board activities to multiples, factors, prime & composite numbers, sequences & algebraic rules

# Who are we?

- Look at the graphs about us.
- What do you notice?
- As a table, write a single sentence that best describes the whole group.

# Analyzing Mathematical Tasks

“There is no decision that teachers make that has a greater impact on students’ opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.”

Lappan & Briars, 1995

# What are Mathematical Tasks?

We define mathematical tasks as a set of problems or a single complex problem the purpose of which is to focus students' attention on a particular mathematical idea.

# Why Focus on Mathematical Tasks?

- Tasks form the basis for students' opportunities to learn what mathematics is and how one does it
- Tasks influence learners by directing their attention to particular aspects of content and by specifying ways to process information
- The level and kind of thinking required by mathematical instructional tasks influences what students learn





# Professional Practice Norms

- Listening to and using others' ideas.
- Adopting a tentative stance toward practice -- wondering about the rationale/outcome for other's professional decisions instead of espousing certainty and being judgmental about what the teacher was thinking or what you believed should have happened.
- Backing up statements with evidence and providing reasoning.
- Talking with respect yet engaging in critical analysis of teachers and students portrayed.



# Comparing Two Mathematical Tasks



# Comparing Two Mathematical Tasks

## Solve Two Tasks:

⌘ Martha's Carpeting Task

⌘ The Fencing Task

# Comparing Two Mathematical Tasks

## Martha's Carpeting Task

Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

Stein, Smith, Henningsen, & Silver, 2000, p. 1

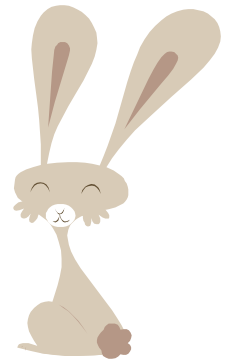


# Comparing Two Mathematical Tasks

## The Fencing Task

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits.

1. If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?
2. How long would each of the sides of the pen be if they had only 16 feet of fencing?
3. How would you go about determining the pen with the most room for any amount of fencing? **Organize your work so that someone else who reads it will understand it.**



Stein, Smith, Henningsen, & Silver, 2000, p. 2

# Comparing Two Mathematical Tasks

How are Martha's Carpeting Task and the Fencing Task the same and how are they different?

*(Consider your own experience in solving the tasks and the “mathematical possibilities” of the tasks.)*



# Comparing Two Mathematical Tasks

## Martha's Carpeting Task

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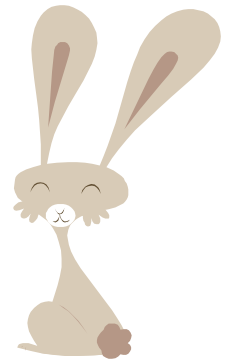


# Comparing Two Mathematical Tasks

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# Comparing Two Mathematical Tasks

How are Martha's Carpeting Task and the Fencing Task the same and how are they different?

*(Consider your own experience in solving the tasks and the “mathematical possibilities” of the tasks.)*

# Comparing Two Mathematical Tasks

“Not all tasks are created equal, and *different tasks will provoke different levels and kinds of student thinking.*”

Stein, Smith, Henningsen, & Silver, 2000

# Comparing Two Mathematical Tasks

“The level and kind of thinking in which students engage determines what they will learn.”

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

# Characterizing Tasks



# Characterizing Tasks

## Goals:

Identify characteristics of high and low level mathematical tasks.

- Martha's Carpet: Low level
- The Fencing Task: High level

# Characterizing Tasks

- Sort the given Tasks into two categories: High level and Low level.
- Develop a list of criteria that describe in general the characteristics of Low level and High level tasks.





# Levels of Cognitive Demand & The Mathematical Tasks Framework



# Linking to Literature/Research

The QUASAR Project

## ■ Low-Level Tasks

- Memorization
- Procedures without Connections (e.g., Martha's Carpeting Task)

## ■ High-Level Tasks

- Procedures with Connections
- Doing Mathematics (e.g., The Fencing Task)



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# Categorizing Tasks

- Are all high-level tasks the same?  
[Is there an important difference between Tasks I and J?]
- Are all low-level tasks the same?  
[Is there an important difference between Tasks A and E?]

# Categorizing Tasks

“If we want students to develop the capacity to think, reason, and problem solve then we need to start with high-level, cognitively complex tasks.”

Stein & Lane, 1996

*Learning by Doing<sup>®</sup>*



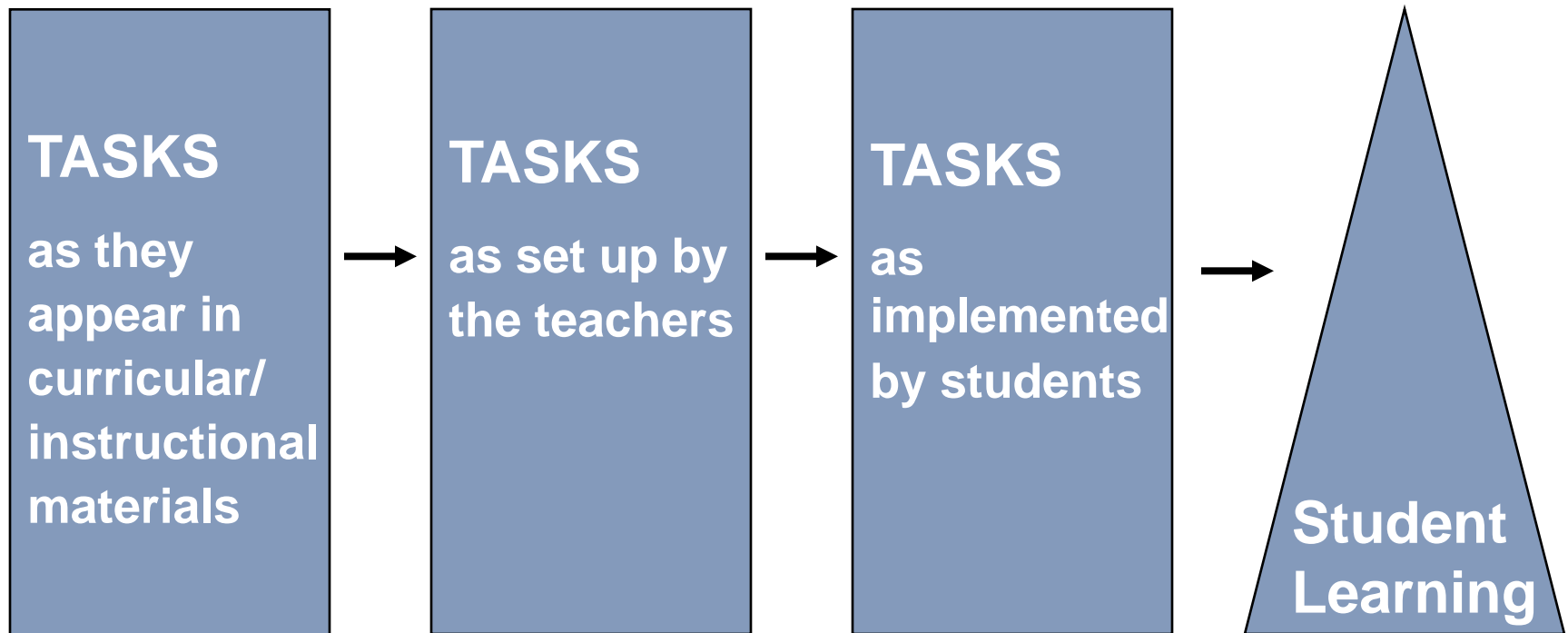
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# Linking to Literature/Research

The QUASAR Project

## The Mathematical Tasks Framework

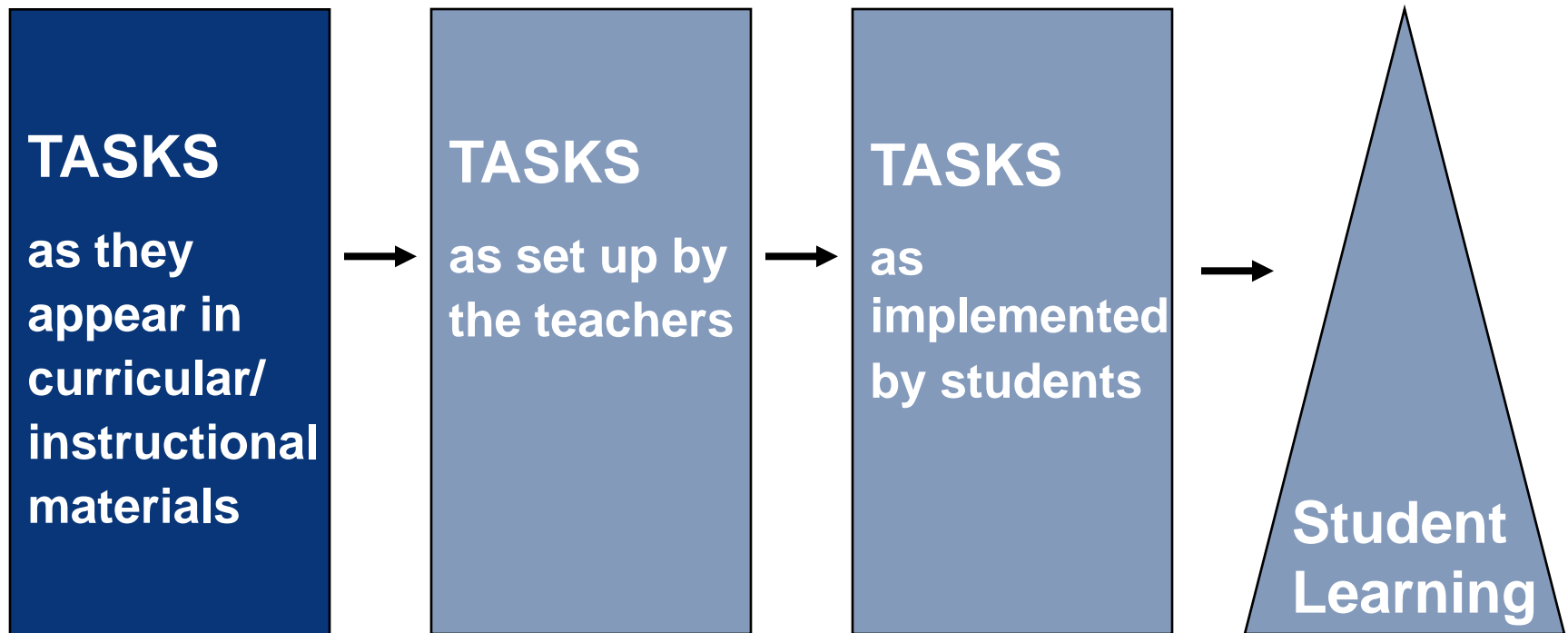


Stein, Smith, Henningsen, & Silver, 2000, p. 4

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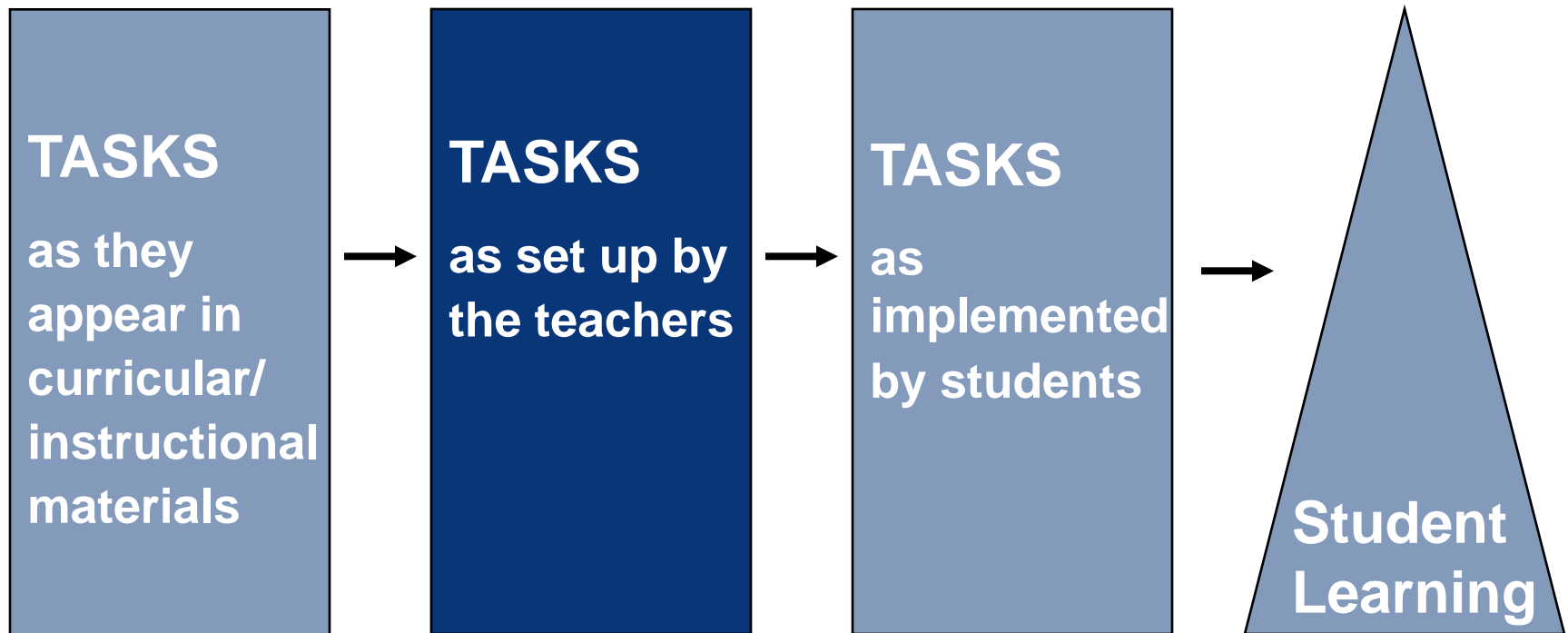


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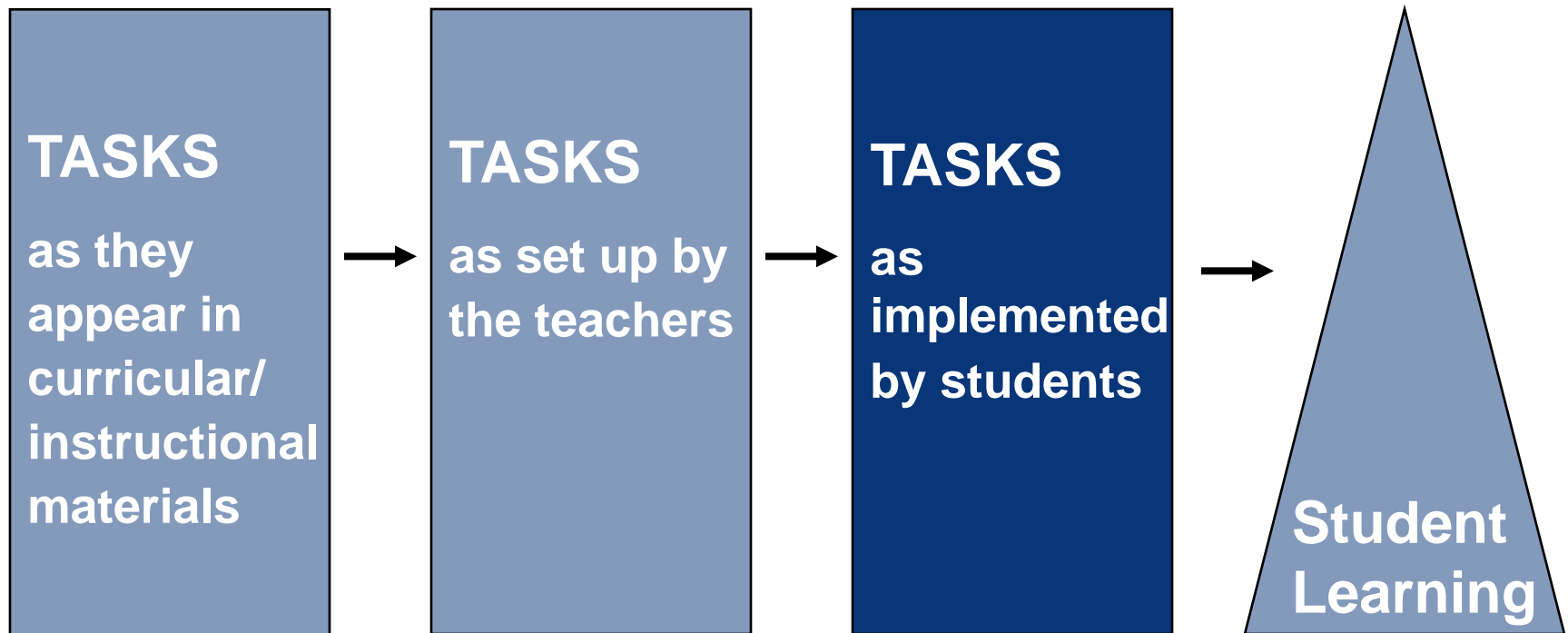


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# Linking to Literature/Research

The QUASAR Project

## The Mathematical Tasks Framework



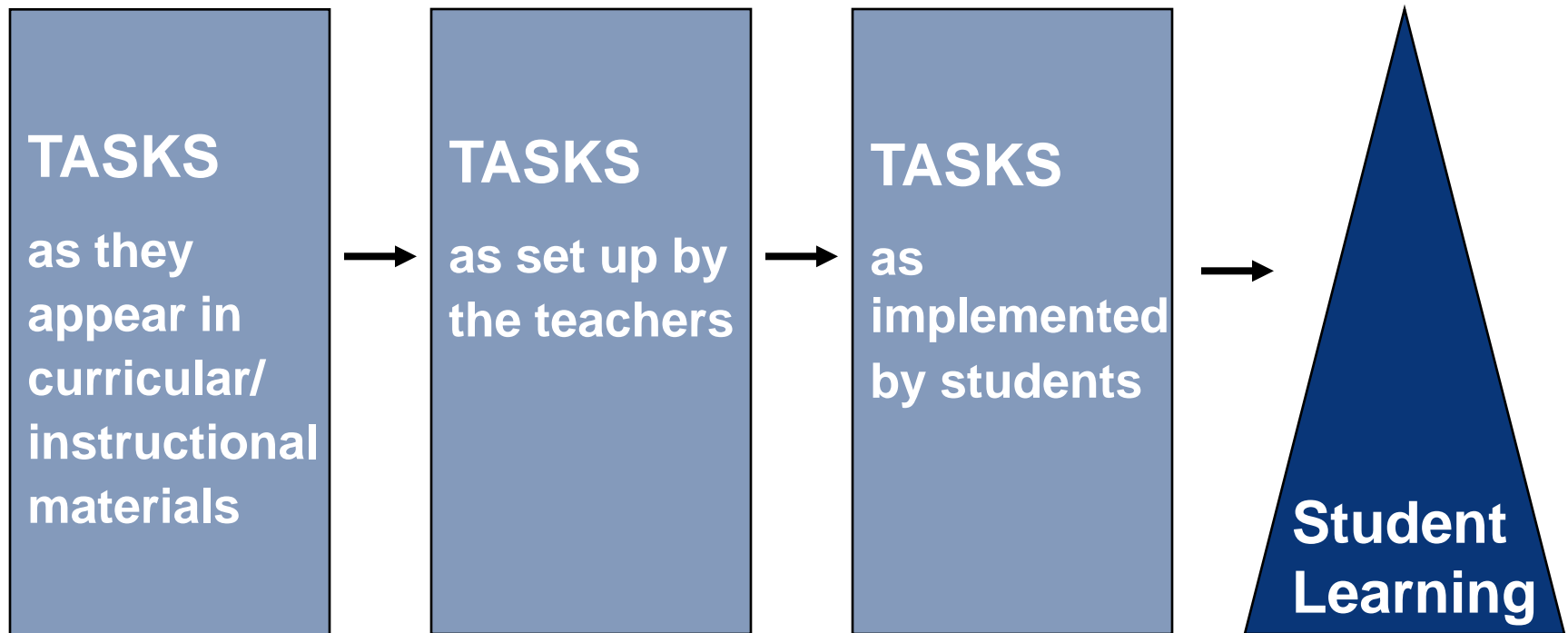
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# Linking to Literature/Research

The QUASAR Project

## The Mathematical Tasks Framework



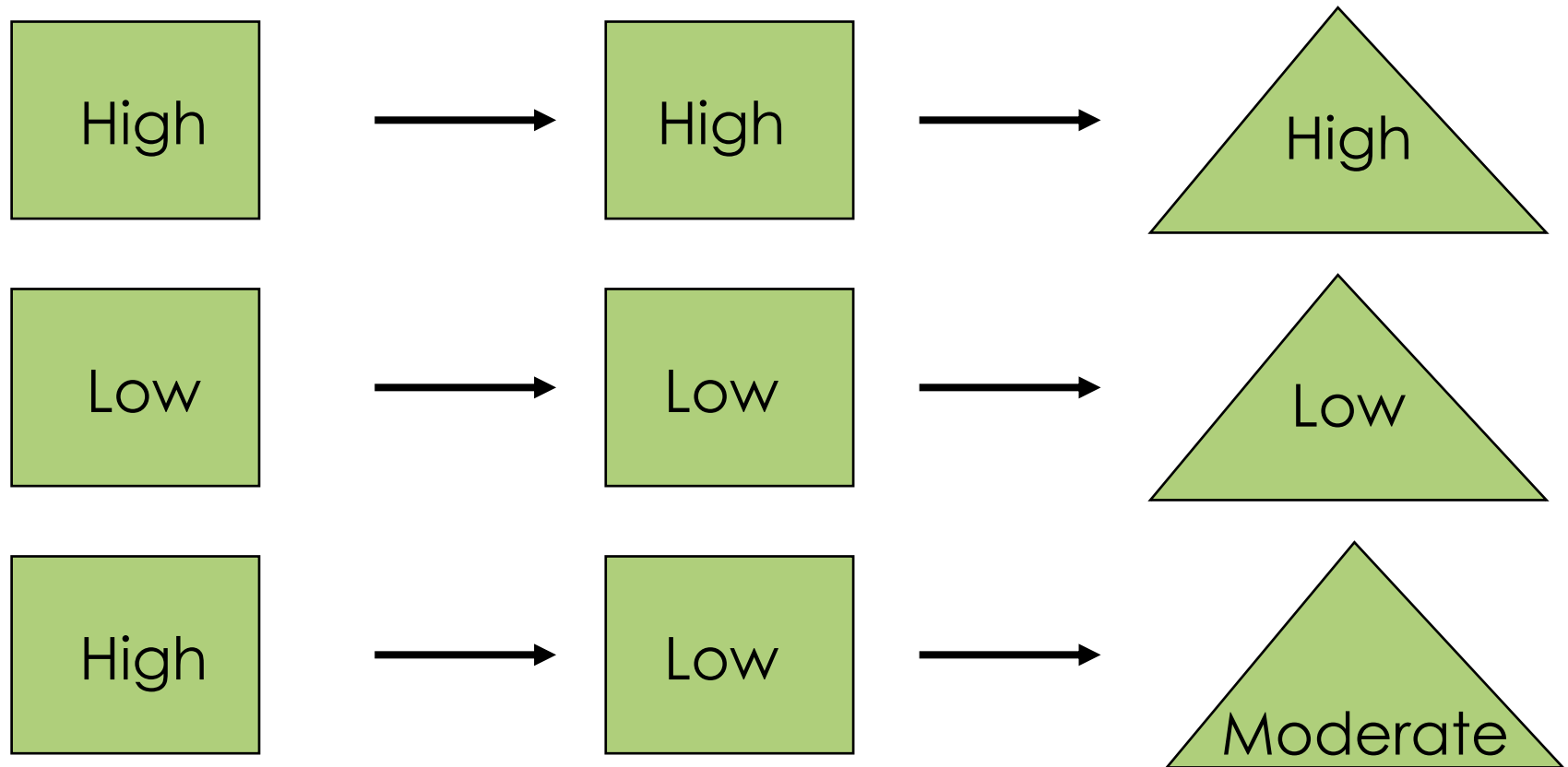
Stein, Smith, Henningsen, & Silver, 2000, p. 4

# Linking Task Level to Student Achievement

**Task Set-Up**

**Task Implementation**

**Student Learning**



Stein & Lane, 1996

# Linking to Research/Literature

The instructional tasks teachers' select are crucial in helping students make connections and learn important mathematics concepts.

Tasks that engage students in thinking about the defining characteristics of important mathematical concepts help students develop a deep understanding of core mathematical ideas that increase retention and transfer of knowledge to new situations.



# Linking to Research/Literature

“Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.”

NCTM, 2000, p.16

# Reflection

Please individually answer the questions below on an index card :

*What implications may the experience of the Task Sort have on my teaching?*

*What and how can I share important pieces of the Task Sort back on my campus?*

# Hundred Board Activities

- Explore Hundred Board Activities across grade levels 3, 4 and 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# Hundred Board Patterns

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



# Hundred Board Patterns

- What patterns do you notice in the Hundred Board?
- In your patterns, how are the values of the numbers changing?
- Share your patterns at your tables, be ready to share patterns with the whole group.



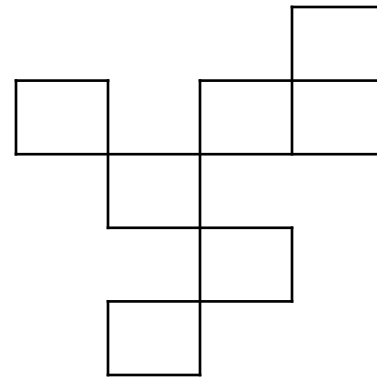
# Eighty Board Patterns

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64
65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80

# Fifty Board Patterns

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50

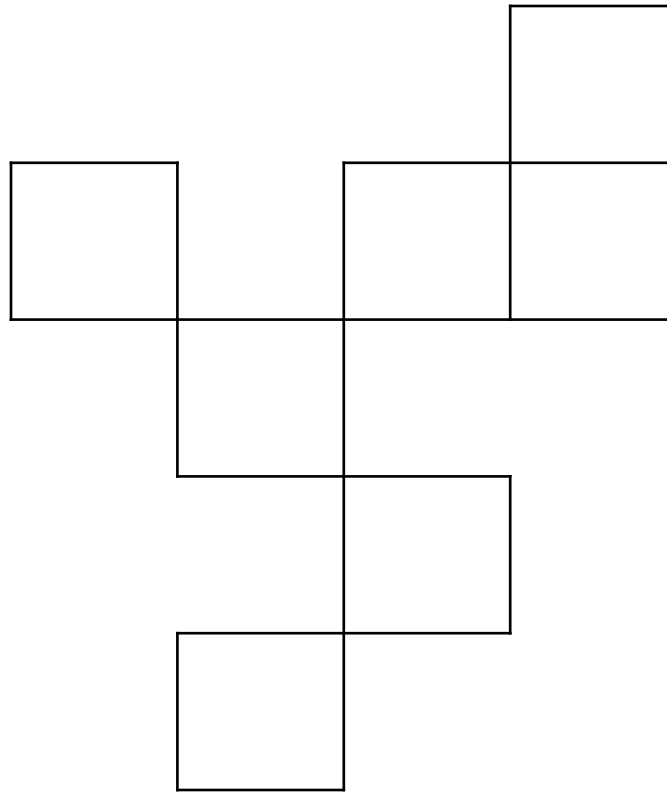
# Missing Pieces



- Complete your “Missing Pieces” without using your Hundred Board
- Find the other participants in the room that have the same **completed** piece of the Hundred Board.
- Discuss and compare with your new group how you completed the Missing Pieces. Be ready to share with the class

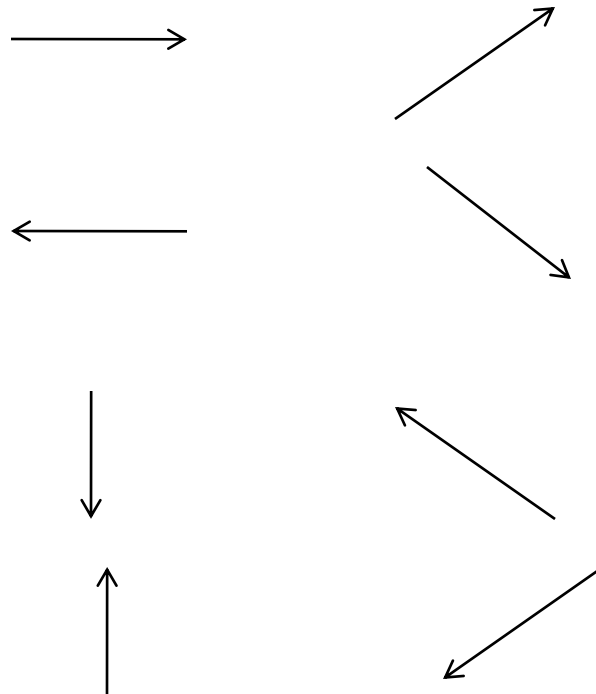


# Missing Pieces



# Hundred Board Arrow Paths

1) Explain how each of these arrows changes a number on a Hundred Board.



# Hundred Board Arrow Paths

2) Create 2 different arrow paths that start at 12 and end at 46. Draw the arrows and each number it takes you to. Then below, write the number sentence that represents your arrow path.

12\_\_\_\_\_46

Number Sentence:\_\_\_\_\_

12\_\_\_\_\_46

Number  
Sentence:\_\_\_\_\_

# Hundred Board Arrow Paths

3) Create 2 different arrow paths that start and end at 78 so that on a Hundred Board the paths form either a triangle, parallelogram, pentagon or hexagon. (Each arrow path must create a different shape)

78 \_\_\_\_\_ 78

# Hundred Board Arrow Paths

## Grade 4

4) Create an arrow path so that on a Hundred Board the path forms your favorite letter of the alphabet.

Starts at \_\_\_\_\_

\_\_\_\_\_

Ends at \_\_\_\_\_





# Hundred Board Arrow Paths

5) Draw every possible pair of arrows that make NO change to a number.

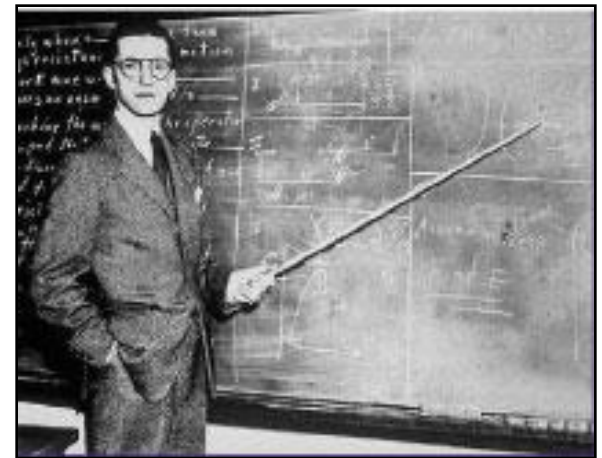
# Hundred Board Arrow Paths

6) Does  $\uparrow \swarrow$  make the same change to a number as  $\swarrow \uparrow$  ?

Justify your answer with numeric examples from the Hundred Board.

# Looking through Teacher Lenses

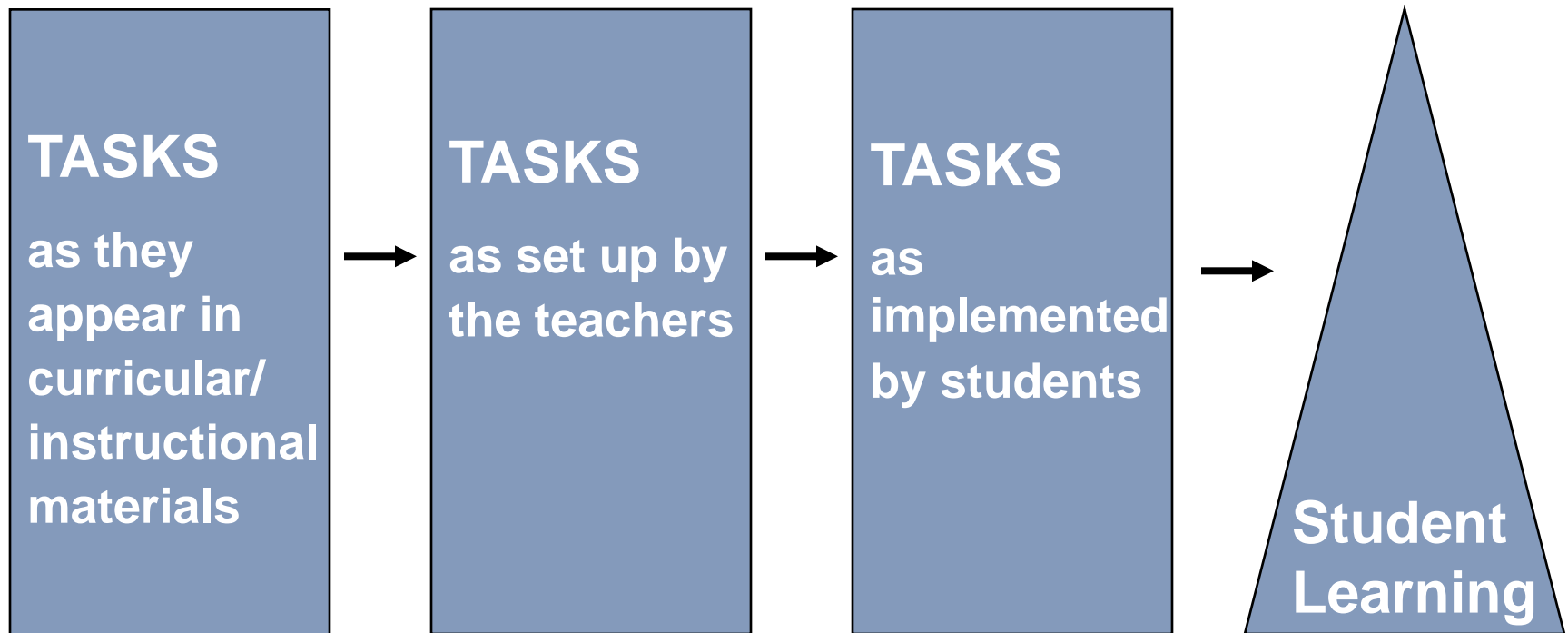
- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
- What makes this worthwhile mathematics?



# Linking to Literature/Research

The QUASAR Project

## The Mathematical Tasks Framework



Stein, Smith, Henningsen, & Silver, 2000, p. 4

# Exploring Multiples

- You will begin at all the following numbers below on a Fifty Board. Underline the first multiple, then put a colored dot in all **SUCCEEDING** multiples of that number.
- Multiples of 2 = Red
- Multiples of 3 = Green
- Multiples of 4 = Blue
- Multiples of 5 = Purple
- Multiples of 6 = Yellow
- Multiples of 7 = Orange
- Multiples of 8 = Black
- Multiples of 9 = Brown
- Multiples of 10 = Gray
- Multiples of 11 = Turquoise
- Multiples of 12 = Pink

# Start and Jump Numbers

Start with 3 and skip count by 5s

Step #	# in Pattern
1	3
2	
3	
4	
5	
6	
7	
8	
9	
10	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# Start and Jump Numbers

Start with 6 and skip count by 5s

Step #	# in Pattern
1	6
2	
3	
4	
5	
6	
7	
8	
9	
10	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# Start and Jump Numbers

Start with 3 and skip count by 4s

Step #	# in Pattern
1	3
2	
3	
4	
5	
6	
7	
8	
9	
10	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
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71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



# Start and Jump Numbers

Start with 5 and skip count by 4s

Step #	# in Pattern
1	5
2	
3	
4	
5	
6	
7	
8	
9	
10	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
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51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# Start and Jump Numbers

- What do you notice when only the start number changes but the jump pattern remains the same?
- What do you notice when the start number remains the same but the jump number increases? *(starting at 3, but skip counting by 4s, then by 5s)*

# Sequences

5, 8, 11, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

Can you find the 100<sup>th</sup> term in this sequence? Can you write a rule to find the ***n***th term in the sequence?



# Sequences

Use a table to help find the 100<sup>th</sup> term and the ***n***th term of the sequence.

Step	Number in Pattern
1	5
2	8
3	11
4	
5	
6	
10	
100	
n	

# Reflecting on Day 1



- Solve, analyze, and discuss mathematical tasks
- Consider the effects of different levels of mathematical tasks on students' achievement
- Represent problems using multiple representations
- Explore Hundred Board activities across grade levels 3, 4, and 5
- Utilize the Hundred Board to understand prime and composite numbers